Means, modes and medians

What are the average heights, most common heights and the middle height values for males and females aged 13 years living in Queensland, Australia?

Let's use the Queensland *CensusAtSchool* data to find out.

The **Mean (or Average)**

The **mean** or average is a measure of the centre of the data. Adding all the values together and dividing by the total number of values finds it.

For example the mean (or average) height of 7 boys whose heights are
140 cm, 140cm, 150cm, 150 cm, 163cm, 163 cm and 163 cm   is :

$$\frac{140 + 140 + 150 + 150 + 163 + 163 + 163}{7} = 152.7 \text{ cm}$$

You will notice that 2 boys each had a height of 140 cm. Adding 140 twice is the same as multiplying 140 by 2.
So we could have found the mean height by multiplying 140 by 2, 150 by 2 and 163 by 3, adding these answers together and then dividing by 7.

140	X	2	=	280
150	X	2	=	300
163	X	3	=	489
Total				1069
Mean Height		1069/ 7	=	**152.7**

The table below shows height categories or ranges of 13 year old boys and the number of boys in each range.

105.5*1=105.5

Males 13

Height	midpoint (x)	frequency (f)	Cumulative frequency	midpoint * frequency (x*f)
101-110	105.5	1	1	105.5
111-120	115.5	1	2	115.5
121-130	125.5	6	8	753.0
131-140	135.5	36	44	4,878.0
141-150	145.5	221	265	32,155.5
151-160	155.5	488	753	75,884.0
161-170	165.5	397	1,150	65,703.5
171-180	175.5	119	1,269	20,884.5
181-190	185.5	20	1,289	3,710.0
191-200	195.5	0	1,289	0.0
Total		1,289		204,189.5

But- we do not know the exact heights of the students! We only know their height range. So how can we calculate a mean height? We approximate and say that every student in a particular height range has a height that is equal to the midpoint of the range. **Will this always be true?**

How do we calculate the midpoint of the height range? Look at the table for Males. The midpoint for males of height 101 – 110 cm is found by adding the two end points of the range together and dividing by 2. (101 + 110 = 211 / 2 = 105.5)

The frequency is the number of times a value occurs. For example, there are 6 males aged 13 whose height is between 121 and 130 cm.

Multiply the frequency (f) by the midpoint (x) and place the answer in the frequency * midpoint column. (6 x 125.5 = 753.0)

Find the total of the frequency x midpoint column.

The **mean** (or average) is then calculated by dividing the total of the frequency x midpoint column by the total number of boys (the sum of the frequency column). So for males the mean is $\dfrac{204{,}189.5}{1{,}289} = $ **158.4.**

Now here is a table for Females aged 13. Fill in the columns you need and calculate the mean height for females aged 13 years.

Females 13

Height	midpoint (x)	frequency (f)	Cumulative frequency	midpoint * frequency (x*f)
101-110		1		
111-120		1		
121-130		9		
131-140		25		
141-150		200		
151-160		639		
161-170		504		
171-180		82		
181-190		1		
191-200		1		
Total		1,463		

The **mode** is simply the value (in this case height) than occurs most often. Because our data is shown in ranges, we can talk about the modal range. For males, the mode (or modal range) is the range 151-160 cm because this range has the highest frequency of 488. Work out the mode for females.

The **median** is the 'middle' value of the data. If you were to make the boys line up in order of height, the median height would be the height of the boy in the middle. There would be as many boys to the left of the boy in the middle as there were to the right of the boy in the middle. Let's calculate the median height for males.

The total number of males is 1,289. Which male student would be in the middle if they were lined up in order of their height?

Can you fill in the table below?

Number of Students	Middle Student
3	2nd student
5	3rd student
7	4th student
	10th student
1289	

Can you see a pattern?

Yes, that's right, the position of the middle value is found by adding 1 to the number of students and dividing this by 2.

In our data, the middle boy is number 645. (1289 + 1 = 1290 / 2 = 645)

How can we easily find the height range for boy number 645? We use something called the 'cumulative frequency'. The cumulative frequency for a particular row is a 'running total' and can be calculated by adding the frequency in that row to the total frequency from all rows before.

Using the cumulative frequency column for the males' table we see that the 645th value is in the height category of 151-160 cm. Therefore the median height is 151-160 cm.
Find the median height for females.

To think about!
- What if the total number of males was an even number? Is there a definite middle position?
- How might you calculate the median in this situation?

Now use the data for the UK CensusAtSchool to work out the average heights for males and females aged 13 living in the UK.

Male frequency	Height (cm)	Female frequency
8	101 - 110	7
10	111 - 120	5
24	121 - 130	24
60	131 - 140	56
441	141 - 150	430
1206	151 - 160	1745
1135	161 - 170	1490
453	171 - 180	216
48	181 - 190	11
11	191 - 200	2
3396	Total	3986